**Exam. Code : 211003**
**Subject Code : 4864**

**M.Sc. (Mathematics) 3rd Semester**

**STATISTICS—I**

**Paper : MATH-577**

Time Allowed—Three Hours] [Maximum Marks—100

**Note :**—Candidates are to attempt **FIVE** questions, **ONE** from each Section. **Fifth** question may be attempted from any Section. All questions carry equal marks.

## SECTION—A

1. (a) Prove that for any discrete frequency distribution, standard deviation is not less than mean deviation from mean.

   (b) Define skewness and kurtosis. How can you broadly classify distributions according to these features ?

2. (a) Define conditional probability. If A and B are two events with $P(A) = 0.6$, $P(A \cap B) = 0.3$ and $P(B) = 0.5$, find the values of $P(\overline{A}/\overline{B})$ and $P(A/\overline{B})$.

(b) Let X has the probability density function

$$f(x) = \begin{cases} c + x & ; \quad -2 < x \le 0 \\ c - x & ; \quad 0 < x \le 2 \\ 0 & ; \quad \text{otherwise.} \end{cases}$$

Determine the constant c, cumulative density function F(x) and find $P\left(-\frac{1}{3} < x \le \frac{4}{3}\right)$.

## SECTION—B

3. (a) For any two random variables X and Y, prove the following :

(i) E(X) = E[EX/Y]

(ii) U(X) = E[U(X/Y)] + U[E(X/Y)].

(b) Let X and Y be jointly distributed with p.d.f.

$$f(x, y) = \begin{cases} \frac{1+xy}{4} & , \quad |x| < 1, |y| < 1 \\ 0 & , \quad \text{otherwise.} \end{cases}$$

Are X and Y independent ?

4. (a) Define joint, marginal and conditional prob function of random variables.

(b) Suppose that (X, Y) is a two dimensional random variable with joint p.d.f.

$$f(x, y) = \begin{cases} 2(x+y-2xy) & , \quad 0 \le x \le 1, 0 \le y \le 1 \\ 0 & , \quad \text{otherwise.} \end{cases}$$

Find the marginal density function of X and Y.

## SECTION—C

5. (a) Let the random variable X has p.d.f.

$$f(x) = \begin{cases} \dfrac{1}{2\sqrt{3}} & , \quad -\sqrt{3} \le x \le \sqrt{3} \\ 0 & , \quad \text{otherwise.} \end{cases}$$

Find an upper bound to the $P\left[|X| \ge \frac{3}{2}\right]$ using Chebychev's inequality and compare it with the exact probability.

(b) Define convergence in probability. Let $X_1$, $X_2$, ....., $X_n$ be iid random variables each following the distribution :

$$f(x) = \begin{cases} e^{-(x-\theta)} & ; \quad x \ge \theta \\ 0 & ; \quad \text{otherwise.} \end{cases}$$

Show that $\overline{X}_n$ converges in probability to $(1 + \theta)$.

6. (a) What is Hypergeometric distribution ? Find its mean. How is it related to binomial distribution ?

(b) State and prove the central limit theorem for independent and identically distributed random variables.

## SECTION—D

7. (a) Define Beta distribution of second kind. Obtain its mean and variance.

(b) Show that exponential distribution 'lacks memory'.

8. (a) Given X = 4Y + 5 and Y = KX + 4 are the lines of regression of X on Y and Y on X respectively.

Show that $0 < 4k < 1$. If $k = \frac{1}{16}$, find the mean of the two variables and coefficient of correlation between them.

(b) What is association of attributes ? Write a note on the strength of association and how it is measured.